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Mozart, metaphor and math

In which we watch Salieri "listen" to Mozart's manuscripts, and we wish it were as easy to show the beauty within equations. Then, since equations so exquisitely encapsulate understanding, we reflect on other forms of compressed knowledge — like formulae, metaphors, parables and graphs.

In a wonderful scene from the movie, *Amadeus*, Mozart's wife, Constanze, surreptitiously brings a pile of her husband's compositions to Court Composer Antonio Salieri. Her objective is to secure a commission for her husband Wolfgang. By showing Salieri the compositions she hopes to demonstrate Wolfgang's skills and thereby persuade Salieri to appoint her husband tutor to the emperor's niece. Of course she accomplishes no such thing. Because as Salieri reads Mozart's compositions the music fills his head — and jealousy his heart. A cinematic trick brings the scene alive. Knowing that few of us in the audience can read music, the director doesn't film Mozart's manuscripts. Instead, the camera zooms to Salieri's face as a *music-over* envelops us within Mozart's genius — and helps us understand Salieri's pain.

I wish there were some analogous cinematic trick to show the beauty in equations. Maxwell's equations of electromagnetic fields, for example[1]. Rather than a music-over, could we imagine a *physics-over*? How about a physics-over showing the equations used as threads to spin a cocoon? Within the cocoon, a butterfly of electromagnetic waves takes shape. Might the camera zoom to this cocoon as it slowly opens to release its butterfly — a butterfly of electromagnetic waves?

Before the cocoon opens, we don't know what *species* of butterfly will emerge. Could the butterfly be radio waves arriving from a Mars probe? Or moonlight reflected from an undulating ocean? Or cell phone signals seeking a tower? Our physics-over cocoon contains the DNA for all of these — and also for sunlight illuminating our world and radio waves linking our civilization. We watch — impatient — until this butterfly unfolds its wings and we see a rainbow.

I've stacked metaphor on top of metaphor. So we'll need a very clever cinematographer. The threads are the four electromagnetic *field* equations. The cocoon is the *wave* equation spun from the field equations. The butterfly is a rainbow — the unique *species* of electromagnetic wave that emerged, this time.

During our odyssey to find the something-of-value we scrape from energy flows[2], you might wonder why we've sailed into this little cove to drop anchor and talk about links between Mozart and equations. I consider this anchorage a place for mid-voyage provisioning — for provisioning perceptions. I want a happy ship. And understanding the role of equations can help bring fair winds and a cheerful crew. Unfortunately, many of our shipmates have learned to abhor equations, just as Captain Cook's crew of HMS *Endeavour* abhorred sauerkraut. Yet the Captain's insistence that sauerkraut be included within a diet of salt pork, dried peas and weevil-spiced biscuits, kept his crew alive and healthy to enjoy the sights, fragrances and pleasures of the South Pacific[3].

Over the next couple of articles we'll use a few equations to help make ideas crisp, or at least crisper. They will be simple equations, because the most sweeping engineering concepts are described by simple equations and these are all we'll need. Besides, I have a big advantage over Mozart. If you compare the manuscript for Mozart's *Mass in C*

[1] In this series of IJHE articles, these equations were discussed in Scott DS. "Engineering and Classical Physics". Int J Hydrogen Energy 2000;25:801–806.

[2] The purpose of our odyssey was set out in Scott DS. "Afternoon on a Hillside", Int J Hydrogen Energy 2000;25:603–4.

[3] Under this otherwise corporal-punishment reluctant captain, refusing sauerkraut was one of the few transgressions that could lead to the lash.

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Minor with the few equations we'll encounter, I expect we'll agree that the equations are easier for most folks to "read".

While talking about a physics-over of cocoons, butterflies and rainbows, I asked you to trust me. I didn't write down Maxwell's field equations and then explain how they gave the wave equation and, in turn, allowed us to understand rainbows. My explanatory skills were not up to the challenge. I just told you how I think about equations — perhaps how I *feel* about them — especially, some equations. So we must dig deeper.

Etymology is one way to start. The obvious roots of "equation" are "equate" and "equal". The *Oxford English Dictionary* says an equation is a "statement of equality"..."which indicate(s) a constant relationship between variables." *Webster's Unabridged* says "the act of equating or making equal"...and..."equally balanced." So the first step in reading an equation is simply to know that the stuff on the left is equal to the stuff on the right.

Of course we must ask: What kind of stuff? Equal in what way? Usually, equal in several ways. For example, the two sides might be equal in energy (if the equation is written to encapsulate nature's law of energy conservation). Or equal in material (if the equation encapsulates nature's law of mass conservation), or equal in momentum, or electrical charge, and so on. In all these examples, the stuff that is equal — the stuff in balance across the equation — has a pedigree going back to the eleven laws of classical physics[4].

I've admitted that it's beyond my skills to describe simply, in words, how the electromagnetic wave equation can help us understand how a rainbow "works". So let's select a different example. Let's choose an equation that describes a *process* most of us know from high school — the process of splitting water into its molecular components. Rather than write an equation for an arbitrary amount of water, we will write the equation in its simplest form — in this case, splitting just two molecules of water into two molecules of hydrogen and one of oxygen. Of course, we can apply the equation to any amount of water by simply multiplying the two sides by an appropriate constant. Now we have:

$$2H_2O \rightarrow 2H_2 + O_2 \tag{1}$$

This water-splitting equation is written so the two sides are equal in material. It requires that the amount of material in the water entering equal to the amount of material in the hydrogen and oxygen leaving. The left hand side describes the mass that goes *in*: water (H_2O). The right hand side describes the mass that comes *out*: hydrogen (H_2) and oxygen (O_2). Yet the equation gives still more information. Because it also requires that the number of hydrogen *atoms* (H) entering must equal the number leaving. Four go in (the subscript means two and the coefficient means two times two), and four come out. Of course, the equation also requires an atomic balance for oxygen: two atoms go in and two come out.

Experienced equation readers will know that our Eq. (1) can also be used to balance energy as well as material. But if we want to include energy, we will need to mentally "read in" now-invisible energy terms. To help remind us to balance energy, we might re-write the equation as,

$$2H_2O + \text{energy} \rightarrow 2H_2 + O_2 \tag{2}$$

Of course if we are going to use Eq. (2) as an energy balance, we must remember that there can be energy stored in the material. Therefore, the material terms, like H_2O, might also represent energy.

Let's move from the *general* water-splitting Eqs. (1) and (2) to the *specific* process of electrolysis. For electrolysis, we can write a combined energy and material equation as:

$$2H_2O + \text{electricity} \rightarrow 2H_2 + O_2 + \text{waste heat} \tag{3}$$

Eq. (3) includes all energy forms. Therefore, in addition to the input electricity and output waste heat, we must account for any energy that enters or leaves via material. Obviously, the output hydrogen carries chemical energy. But the input water and output oxygen might also contain energy if, for example, they are at elevated temperatures

[4] See Reference of Footnote 1.

or pressures. Elevated temperatures or pressures mean the material is out-of-equilibrium with the environment — and, therefore, contains "energy" for which we must account. This energy is best called "thermomechanical" energy[5].

So now the input energy is the sum of the electrical energy plus any thermomechanical energy carried by the water. The output energy is the sum of waste heat, plus the chemical energy carried out with the hydrogen, *plus* any thermomechanical energy carried by the hydrogen and oxygen. Eq. (3) accounts for all forms of energy, whether chemical, electrical or thermomechanical, and as inputs or outputs.

What about the material balance? Well the material balance is the same as we had for Eqs. (1) and (2). Nothing new.

So now Eq. (3) is a crisp, compact encapsulation of the mass and energy conservation laws for electrolysis. Using the equation is simply a matter of careful addition and subtraction — just a matter of accounting. No fancy math.

Any of Eqs. (1), (2) or (3) correctly describe electrolysis. It's choice, not memory. Your choice should be one of convenience — and should depend on what aspects of the process you want to illuminate.

I never memorize equations. I try to understand the physics and then write what is, for me, a convenient mathematical visualization to describe the physics. It comes back to thinking of equations as describing phenomena we already understand and, through the equation's precision and compaction, can come to understand much better. Equations both encapsulate understanding and lead to deeper understanding[6].

Of course just because we can write water-splitting equations does not mean that every time we mix water and energy the result will be hydrogen and oxygen. For example, if we mix water and energy by heating a pot of water on an electric stove, we'll simply get a pot of hot water — and probably some steam. So our equations do not tell us what *always* happens when we add energy — even in the form of electricity — to water. Rather they tell us what *could* happen — and will happen if we choose a specific process like electrolysis.

Commenting on this my wife said, "Now you're telling me I can write down anything I want and it will be O.K." Not quite. If you watch Sally kick a red ball into a busy street, you could say "A red ball kicked by Sally just bounced into the street." Or, "Sally just kicked her red ball into the street." Or, "I hope Sally sees the car coming down the street if she decides to chase the red ball she just kicked into the street." The first two sentences correctly describe what has happened. The third sentence correctly describes what has happened and hints at what you hope won't happen. It's a matter of how you want to present the facts. But if you say, "A car playing in our yard just threw a green ball into the street and the ball might hit Sally now skipping down the sidewalk" you got it wrong in several ways. And the results could be unfortunate.

Any fool can write an almost infinite number of wrong equations. But that doesn't mean there is only one way to write a correct equation.

While going on about the water-splitting equation,

$$2H_2O + \text{energy} \rightarrow 2H_2 + O_2 \tag{2}$$

I can't resist writing the inverse water-making equation,

$$2H_2 + O_2 \rightarrow 2H_2O + \text{energy} \tag{4}$$

In wonderfully compressed form, this two-equation package expresses the *leitmotif* of the coming hydrogen age. The water-splitting Eq. (2) encapsulates the truth that (by using appropriate technologies) we can take energy from any source and use it to split water into hydrogen and oxygen. The hydrogen product can be stored to be used later

[5] If I'd not included the "out-of-equilibrium" phrase, and simply said "energy", I suspect few readers would stumble. But all material contains energy — whether or not it's out-of-equilibrium with its environment. So what we are really doing is accounting for *that portion of* the material's non-chemical energy that could, ideally, be released by bringing the material into equilibrium with its environment. These ideas will become clearer when our odyssey has arrived at its golden fleece: "exergy". In general, thermomechanical energy can exist, not only if the temperatures or pressures elevated above the equivalent environmental conditions, but also if they were lowered below environmental conditions. To be below environmental pressures or temperatures is very unlikely during electrolysis, but it sure can happen with a cryofuel, like liquid hydrogen (LH_2) — something that will be of much interest during the hydrogen age.

[6] While reading drafts of this article to help me untangle the prose, my daughter, Sue, upon reaching this section: "Dad, I remember when you were trying to explain this to me while attempting to jam an entire calculus course into my head in a period of two days. May I suggest that, under extenuating circumstances, equations are better memorized, or even scrawled on your wrist under your shirt."

as fuel, while the oxygen product can be used for industrial processes — sometimes for environmental clean-up — or thrown out to the atmosphere. The water-making Eq. (4), says that (by using an appropriate technology) we can get the energy back and produce water by combining the hydrogen with oxygen (which is usually drawn from air). The released energy can power cars or fly airplanes, while the water can be given back to the environment from whence it came — or used as drinking water (a slightly more circuitous route back to the environment)[7].

The remarkable thing about this small package of two equations is how it so efficiently encapsulates the closed cycle of hydrogen systems — a closed cycle that lies at the root of hydrogen-age environmental gentility. Once we know how to "read" the equations, they illuminate profound things about our future. If we were restricted to the inefficiency and imprecision of words alone, it would be impossible to gain these insights with the precision, brevity and numeracy that the equations allow.

About now, remembering your high-school chemistry class you might ask, why don't we make things even more efficient by using just one equation with two arrows — one aiming to the right, the other to the left? Chemists frequently use the dual-arrow formulation to show how changing the external conditions can drive a process forwards or backwards. But for the H_2 age, the water-splitting equation applies where the hydrogen is *manufactured* — analogous to today's gasoline refinery. Conversely, the water-making equation applies where the hydrogen is *used* — which may be high in the sky powering an airplane, or along a country road powering your car. Manufacturing and using happen at different places and different times. So while you *could* write a single forward and backward equation, this is a case where convenience calls for two equations — because a two-equation format is less likely to mislead.

I introduced our water-splitting, water-making package because it's a lovely example of how equations sometimes give me a flutter of joy. The joy comes from seeing, in overview, how something works. In this case, it is an overview of our coming hydrogen age and the underlying reason for its environmental gentility.

I find irresistible, "*j'ecris pour savoir ce que pense*" — loosely translated as "I write to better understand that which I ponder"[8]. Then there is Francis Bacon's observation, "Reading maketh a full man; conference a ready man; and writing an exact man." Equation writers set down equations that encapsulate what they know, or are beginning to guess, and then use the equation to learn much more about what they know, or are guessing. So equations are used in at least three ways:

- to test the truth of what you've guessed;
- to understand more clearly what you know;
- to use what you know to learn things you don't yet know.

Since equations so exquisitely encapsulate knowledge, it can be fun to compare them with other tricks we use to encode and compress understanding — sometimes wisdom. Let's look at a few descriptors that, compared to equations, seem, if not siblings at least cousins.

I'll start by comparing equations with formulae. We value equations for at least three reasons:

- they are an extraordinarily efficient way to encapsulate how things and nature tick;
- by mathematical tweaking, they allow us to learn more about how things and nature tick;
- they can be used to precisely calculate how things and nature will tick in the future.

Formulae are very different beasts. A formula might be used for mixing pigments in a paint store to achieve a special color for your living room, or mixing flours to make bread. There is nothing elegant or graceful about a formula. It tells you how to get a certain color — but not a grand principle for *all* colors. Similarly a formula (recipe) for bread does not give a general, powerful insight about all foods, or even all breads, it just tells you how to make one kind of bread. And if the recipe is used with a little inaccuracy, the result won't be much different than if the baker had achieved perfect accuracy.

So equations and formulae live in different worlds. Equations are elegant, general, and powerful tools for crisp description and precision analysis. The word "formula" has no lineage back to a word like "equal" — there is no exquisite balancing. They are blunt, application-specific workhorses. They might give you a rule for how to mix white, red and brown paint to achieve a rusty hue — but never an insight into painting, even painting living rooms. And certainly never an insight into Van Gogh's artistry.

[7] In space vehicles, fuel cells make electricity to power technical systems and water for the astronauts or cosmonauts to drink.
[8] I haven't been able to identify the author and would welcome help from IJHE readership.

Equations float above in a different league. Because equations *can* give wonderful insights into nature's artistry, like nature's rainbows.

The "core business" of metaphor — I think of a parable as an extended metaphor — is to store truth in compressed speech. So metaphors share with equations the ability to encapsulate ideas. But unlike equations, metaphors *eschew* precision. For as Lawrence Joseph wrote in *Gaia: the growth of an idea*[9], "Metaphors are like myths, often certifiably false yet purportedly chock-full of veracity and wisdom." Sometimes the imprecision of a metaphor can be one of its strengths. There are times for photographers and times for impressionists. Yet the softness, the imprecision, is also why metaphors and parables are vulnerable to "yabut" attacks — yabut it doesn't hold in this or that circumstance — yabut the story didn't really happen that way. We should recognize the yabuts but not, necessarily, allow them to discredit the *idea*.

Even if the details of a parable are certifiably wrong — for example, even if there had never *been* a cabin boy on Admiral Shovel's ship, let alone a cabin boy who was hanged for his curious habit of sniffing the breeze for a hint of land — a parable can still encapsulate truth. Or, considering a parable from a well-known book: Can we really *prove* there was a Samaritan walking down the road just in time, having the time and taking the time to aid a mugged Israelite lying in the ditch? Was Jesus ever asked to *prove* his story? Or do we think Jesus just made it up? Is the *message* of the parable weakened or strengthened by whether it was, or was not, a precise factual record? Or do we think Christ simply knew that, at one time or other, there *must* have been a Samaritan that helped a bleeding Israelite?

Metaphors are useful because their encapsulation gives a toehold for understanding — a toehold that can lead to the next level of understanding as we peck away at the ragged, soft, fuzzy edges. The cliché, "the exception proves the rule" speaks to pecking at the edges. Still, a metaphor sits at one end of the precision spectrum. They are something squishy — something that doesn't allow too much exactitude.

While equations and metaphors share the ability to encapsulate understanding, they lie at opposite ends of the precision spectrum. Diagrams and graphs lie closer to the middle. Take the equation for the acceleration force law,

$$\mathbf{F} = m\mathbf{a}$$

It says the force on something is equal to the mass times the acceleration of that something — like a rocket.

A graph can be used to plot the *special-case* result when the law is applied to a unique rocket acted on by unique forces. For example, by using this force law we could, in principle, draw the position-time graph for a Saturn rocket liftoff from Kennedy Space Center — *if* we accurately knew the mass and mass-distribution of the rocket and the vector sum of all the forces, like engine thrust, crosswinds, etc. Still a graph will *never* be as accurate as the equation. And never as general. Because a graph can only describe a uniquely defined rocket having a uniquely defined thrust, launched during unique atmospheric conditions.

Conversely, the equation not only works for all rockets under all conditions, it also works for a whole lot more than rockets. Indeed, it describes *and* explains every physical trajectory in our world, from the trajectory of your car, to the flight path of a kicked football, or of a mosquito.

We entered this anchorage to provision our odyssey with a few ideas about the role of equations — and found Mozart, Salieri and movie directors pointing the way up the beach. Now, provisioning complete, we must depart this pleasant little cove, sail out into the ocean and continue our odyssey.

We clear the headland, haul the wind forward of the beam and crack on for the next waypoint. That waypoint will be nature's law of entropy growth.

This is the fourteenth in a series of articles by

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