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Entropy

In which we find entropy, the key waypoint of our odyssey.

Almost everyone has heard something about entropy. Most associate it with long-term trouble, think it has something to do with our universe gradually winding down—like a mechanical clock winds down. Some believe that the magic of life violates the entropy law—which gives them another reason to believe life *is* magic. Some of these perceptions contain a measure of truth. Some are nonsense. The thought that life violates the entropy law is nonsense. But that is getting ahead of our story.

Entropy is *real.* It's a real property of everything—just as real as the familiar properties of temperature, volume, density and so on. Indeed, entropy is probably more real because it can always be defined whereas, sometimes, properties like temperature and pressure can't. That's because temperature and pressure represent statistical averages of extraordinarily large groups of particles—like the particles (molecules) that constitute a piece of toast. Entropy, on the other hand, reflects the statistical *distribution* of energy and location *among* any group of particles, even very small groups. So entropy is real and defined even when there are just a few particles, as might exist in a cubic meter of deep space somewhere far beyond Pluto.

Just as everything you touch—a keyboard, a tree, or your significant other—has a temperature and volume, they all also have entropy. Or, taking a slightly different slant—a loaf of bread *contains* entropy just as it contains carbohydrates.

But many things are real. So what is so special about the realness of entropy? The uniqueness of entropy comes not so much from what entropy *is* but rather how entropy *behaves*. And to understand how entropy behaves we should return to the eleven laws of classical physics. Ten of these eleven laws are conservation laws. Each is built around some property that is always conserved, like energy. The striking *exception* is the *non*-conservation of entropy—because entropy is continuously produced.

Still, it's unfortunate that the thermodynamicsts who uncovered this profound law came to—*had* to—describe entropy in terms of disorder, rather than its opposite: structure. Unfortunate, because it's structure we want—that living things and our energy system take from energy flows. So structure is what we want to talk about. Yet what we've named is lack-of-structure. It's as if we wanted to describe "beauty" but could only use the word "ugly". Saying "It is a low-ugliness sunset", doesn't quite work, does it?

Nevertheless we're stuck with "entropy". Not because the engineers and scientists who uncovered its behaviour were trying to be difficult. Rather, it's an unavoidable result of the *concept*. Entropy is zero if something is perfectly ordered—but there is no upper limit to disorder. So the entropy *scale* begins at zero—perfect order—and increases as disorder increases. It's a bit like temperature. Zero temperature (on an absolute scale) means as-cold-as-we-can-get, nothing can get colder—but there is no upper limit to hotness.

You might also ask: if entropy is so important why isn't it more familiar? The answer may be that, from childhood, we've been able to put out our hand to "feel" temperature or pressure. Not so for entropy. What's more, we have thermometers for measuring temperature and gauges for pressure, but there is no such thing as an entropometer[1] for measuring entropy. So we haven't grown up with the idea of entropy.

Nevertheless, we can measure entropy. We measure it by knowing the relationships among entropy and other thermodynamic properties like pressure and temperature. As you might expect, these relationships are given by equations. So to determine the entropy of something like the air in a SCUBA tank we measure, say, the air's temperature and pressure and then plug these values into an equation and out pops entropy.[2] Or, if someone else has done the calculations, you can look up entropy values in tables—or software.

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[1] I attribute "entropometer" to James Lovelock.

[2] There are two ways to do this. First, we can use relationships between the *rate-of-change* of thermodynamic properties with respect to each other. This methodology is the most fundamental because it does not require special knowledge of the material. But it does require that we know these rate-of-change interdependencies. The second method requires detailed empirical knowledge of the interdependence between the material's thermodynamic properties—which takes us back to the "constitutive relationships" we talked about in the 12th article of this series, *Engineering and Classical Physics*.[c]

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Before going further, we must identify a common misunderstanding of the entropy law. This misunderstanding is rooted in the old cliché: a little knowledge is a dangerous thing. This time the correct—but incomplete knowledge—is this:

- Entropy is a measure of disorder, lack of structure or, if you like, mushiness.
- Any *real* process (to distinguish it from an imaginary or idealized process) produces entropy and thereby increases disorder.

So far, ok. The confusion comes if we conclude that, because all real processes produces entropy, entropy increases *wherever* it's produced. That's wrong. It's wrong because *entropy can be moved from one place to another*. We can dig entropy out from where it's produced and ship it away to somewhere else—as fast, or faster than it's produced. So the third, often forgotten, reality is this:

- Entropy can be imported and exported.

Good thing too! Take thinking.

Thinking produces entropy in our brain. If our brains couldn't shed this entropy, entropy build-up would progressively turn our brains into mush. Fortunately we *can* export entropy from our brain. Heat departing the top of our head carries entropy with it. (Baldness has advantages.) Blood carries away still more—exports it to other parts of our body where we pitch it out as heat or high-entropy material.

While thinking about brains, let's use ours to contemplate our moist blue planet.

Compared with brains, Earth incessantly produces truly massive quantities of entropy. Earth sheds this entropy by sending it away as cargo within the infrared radiation it continuously pitches into the universe. This continuous entropy export keeps Earth's total entropy content pretty much constant. It is a balance often misunderstood by certain doomsday folks—I think sometimes purposely. Jeremy Rifkin, for example, wrote an exquisitely incorrect book, *Entropy: A New World View* [1] telling us how the entropy law is driving our planet towards mush. That's nonsense.

So far we've talked a lot about how entropy *behaves* but, except for saying that it represents disorder, not much about what entropy *is*.

Many people, even most engineers who use the idea of entropy to design gas turbines or oil refineries, are satisfied to understand entropy just from how it behaves. That's not all bad. You probably feel you know *what* your neighbour is by watching how he or she behaves. Yet a DNA sequence, or a blood-sugar count, will give different insights. So it is with entropy. Some will be satisfied with learning about entropy's behaviour. Others would like to know what it really is.

If you are among the latter, we must now push through some slightly more difficult ideas. Wrong! We must push through several simple ideas. But when they are stacked upon each other, they pile up to some not-so-simple navigation.

So if you want to give the next bit of sailing a pass, don't feel guilty. Remember we're on an odyssey. You don't have to stay on deck for the whole voyage. You can go below decks for a nice cup of hot mocha while those on deck guide our vessel through this little squall. Soon we'll be out into smoother water when you can come up to see what we've found.

To understand what entropy is we need a few building block ideas and a little conceptual mortar.

First we need the idea of *macro*states and *micro*states, and how they are related. Imagine a World Cup football match. The stadium is packed. On one side the crowd wears red shirts and waves red flags. On the other, blue is *de rigueur*. Overhead a blimp looks down upon the scene, broadcasting TV images round the world. The blimp's TV images show the stadium's *macro*state—the crowds in the stands, approximately how many are blue, how many red and so on. In contrast, the *micro*state defines *who* is in *each* seat. There are an enormous number of different ways the individual spectators—the Melissa Dolanskis, Willemien Haesevoets and John Smiths—can arrange themselves in the stadium so it looks the same from the blimp. But from the blimp these differences don't matter, because they all give the same *macro*state. If Melissa Dolanski and Willemien Haesevoet are both in the "blues", and they decide to switch seats with each other, the *macro*state will be unchanged.

Now we'll carry these ideas over to a gas within a container. Let's consider a fixed volume container and a quiescent gas (as seen by us "in the blimp"). The everyday macroscopic properties like temperature and pressure represent the gas *macro*state. To consider the gas's *micro*state, it's convenient to assume that each gas molecule is an extremely small billiard ball so that everything about the molecule is specified by its position and energy. The *micro*state of the gas is now defined by specifying the "state"—that is the location and energy—of *each* gas molecule. [3]

[3] In this paragraph I've pretended the molecules cannot have angular momentum or contain sub-molecular bits and pieces—a fantasy that doesn't affect the core ideas we're developing. Just as, in the previous paragraph, neglecting whether the football spectators were fat or skinny, old or young had little effect on the *idea* of stadium *macro*states and *micro*states.

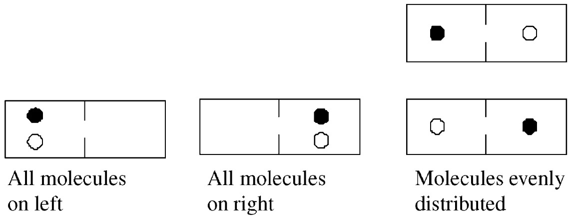

Fig. 1. 2-molecule gas.

Three things should be obvious:

(1) *Each* different location and *each* different energy level of *each* molecule represents a different gas *micro*state. So the *micro*states will change very, *very* rapidly—as molecules bounce off the walls and each other, to whiz hither, thither and yon.
(2) As the *micro*states flash-dance about, the *macro*state remains constant (because the macro-properties like temperature, volume and so forth remain constant).
(3) Therefore, each *macro*state corresponds to an enormous number of different, but rapidly changing *micro*states.

These are our first three building block ideas. We evoked the idea of a football stadium and then a quiescent gas to introduce the meaning of *macro*states and *micro*states. Yet it's important to remember that these ideas will hold for any material in any circumstance—whether a quiescent or flowing gas, or a meteor in the sky, or your toenail.

Let's return to the gas within a container. As these whizzing air molecules come to fill the lobby of your mind, you might wonder what if, by some fluke, they all whizzed up into one corner of the container? If that happened, the *macro*state will no longer be the quiescent gas we assumed earlier. Rather, the *macro*state would be...well...weird! All the molecules rushing into one corner of the container for no reason at all? Still, in terms of our *micro*state idea, there seems no reason why this couldn't happen—because each individual molecule is just as likely to be in one place as another. So, by fluke, they could all be in the same place at the same time. To examine the probability of this "weird" occurrence we'll imagine a *very* dilute gas that is not, necessarily, quiescent. We'll also imagine that a wall, perforated by a hole, divides the container into equal compartments. Let's start with a gas so dilute that it has only two molecules. And, simplifying still further, let's assume the state of each molecule is defined by *only* which side of the container it's in—neglecting its molecular energy, trajectory and other details.

Figure 1 shows that there are three possible *macro*states for our two-molecule gas. The gas can be distributed throughout the container, or it can be all on the right, or all on the left. The key question is: how many *micro*states correspond to each of these three different *macro*states? Two *micro*states correspond to the *macro*state of the distributed gas: molecule #1 can be in the left compartment and molecule #2 in the right, or they can exchange positions so that #1 is in the right and #2 in the left. On the other hand, only one *micro*state corresponds to all the gas in the left side: it's when both molecules are on the left. Similarly, only one *micro*state gives all the gas on the right side.

Next imagine a four-molecule gas. Figure 2 shows that now six different *micro*states all give an *evenly* distributed gas. These six *micro*states correspond to either molecules #1 & #2, or #1 & #3, or #1 & #4, or #2 & #3, or #2 & #4, or #3 & #4 being on right side. For a distributed gas *but* with most of the gas to the left, or most to the right, four different *micro*states correspond to each of these two *macro*states. Finally, there is only one *micro*state corresponding to all the gas on the left—and a different but single *micro*state corresponding to all the gas on the right.

Now we have our fourth building-block idea:

(4) Some *macro*states have myriads of *micro*states. Others have many fewer.

Next, we need some conceptual mortar. At any instant each molecule is bouncing round the container doing its own thing. So there is no reason to claim that any individual molecule is more likely to be in one part of the container than the other—which, using our language, means in one molecular "state" or another. This is the same as saying there is no reason for any *micro*state to be more probable than any other *micro*state. And we have our first dollop of conceptual mortar:

(5) All *micro*states are equally probable.

The second dollop follows from the first because, if each *micro*state is equally probable:

(6) The probability of any *macro*state is proportional to the number of *micro*states that yield the *macro*state.

When applied to our four-molecule gas, this means the probability of finding (at any instant) an evenly distributed gas is six times greater (because there are six *micro*states that give the "even distribution" *macro*state) than of finding all the gas in the left side of the container (a *macro*state for which there is only one *micro*state).

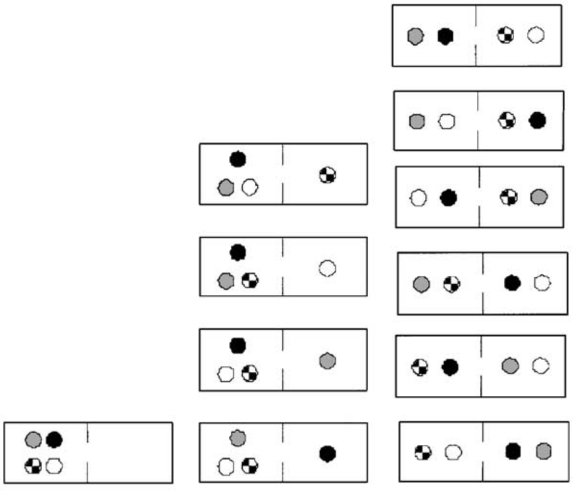

Fig. 2. 4-molecule gas.

If we consider this same container filled but with a real gas there are quad-zillions of molecules whizzing about. So there are quad-zillions of quad-zillions *more micro*states corresponding to these molecules distributed throughout the container than there are *micro*states of all molecules in one compartment. Therefore, the probability that the gas is distributed throughout the container is unimaginably greater than the probability of all molecules spontaneously gathering in one compartment.

But you'd probably guessed as much. So now we must relate all these ideas to entropy.

Entropy only has meaning for *macro*states, not individual *micro*states. Entropy is simply a measure of a *macro*state's probability; it increases as the *macro*state's probability increases. If you like, entropy is a measure of what the stadium looks like from the blimp, not the details of who's in what seat.

What the stadium looks like from the blimp depends on the *macro*state of how the red and blue shirt folks have arranged themselves in the stands. If the stadium's event does not evoke alliances—if there are no competing teams and the entertainment is, say, The Three Tenors—then we'd expect a pretty even distribution of red and blue shirts. It would be extremely unlikely for the *aficionados* to accidentally segregate themselves so that all the blues were one side and all the reds on the other.

How does all this probability stuff relate to structure or disorder? Well! When the molecules are all packed into one compartment of the container, the gas is more structured (less disordered) than when they are scattered throughout. Structure increases when the entropy decreases. Conversely, as something becomes less structured, more disordered, its entropy increases. (Molecules, without alliances to soccer teams, are also without preferences for the container's right or left.)

Let's leave these necessary (but somewhat tedious) ideas about molecular statistics and move on to consider an everyday experience.

The number of different ways that clothes, old newspapers, and yesterday's dishes can be distributed about an unkempt room is much greater than the number of ways that clothes can be folded and carefully placed within their designated drawers, newspapers arranged in a neat pile, and the dishes washed and stacked on their proper shelves in the cupboard. Many more *micro*states yield messy rooms than tidy rooms. The entropy of unkempt rooms is higher than the entropy of tidy rooms.

Still there is a big difference between structure of molecular distributions and structure in your living room. Molecules do not have people stacking them in orderly arrangements. So, unlike the *micro*states of a gas, we cannot claim all *micro*states of your home are equally probable. That's because if you want a tidy home—*and* put in the effort to make it tidy—then tidy *micro*states *are* more likely. "Putting in the effort" is key. It is the effort required to remove the entropy from your room—to move it somewhere else. We'll talk about "where" later.

But imagine your home started out as a mess. The odds that a spontaneous *micro*state fluctuation will come along to rearrange the books, newspapers and dirty dishes so you have a tidy home are zip. And that confirms the entropy law—if we need any more proof.

We've located our entropy waypoint and found that entropy is related to *macro*state probability. It increases or decreases as the probability increases or decreases. High entropy is a symptom of disorder, low entropy a symptom of structure—or, if you like, organization.

Still the idea of entropy remains ephemeral. Not as tangible as temperature or pressure. So before we leave this landfall for the next legs of our voyage let's launch the dinghy, row ashore and tramp about looking for answers to questions like:

(1) Can these ideas be applied to real materials, rather than just to artificial situations like our four-molecule gas?
(2) Since we've only said entropy is *related* to the probability of a *macro*state, is there a precise relationship between the two?
(3) Since the entropy law is probabilistic, what are the odds that entropy could spontaneously decrease?

We'll describe our visit ashore in "The Trouble with *Micro*states" [2].

References

[1] Rifkin J. (with Howard, T.) Entropy: a new world view. Bantam Edition, 1981.
[2] Scott DS. The Trouble with Microstates (17th in this IJHE series). Int. J. Hydrogen Energy, in press.

This is the sixteenth in a series of articles by

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